

Model forms and estimation for linear, log and logit risk GLM models.

| Model Type | Linear | Log Risk | Logit risk |
|----------------------------|---|--|--|
| Model Form | Risk = $\beta_0 + \beta_1 X_1 \dots + \beta_k X_k$ | $\ln(\text{Risk}) = \beta_0 + \beta_1 X_1 \dots + \beta_k X_k$ | $\text{logit}(\text{Risk}) = \beta_0 + \beta_1 X_1 \dots + \beta_k X_k$ |
| Outcome distribution | binomial | binomial | binomial |
| Link function | identity | $\ln()$ | $\text{logit}()$ |
| Simple model | Risk(outcome X_1) = $\beta_0 + \beta_1 X_1$ | $\ln[\text{Risk}(\text{outcome} X_1)] = \beta_0 + \beta_1 X_1$ | $\text{logit}[\text{Risk}(\text{outcome} X_1)] = \beta_0 + \beta_1 X_1$ |
| Risk(outcome $X_1 = 0$) | $R_0 = \beta_0 + 0 * \beta_1 = \beta_0$ | $\ln(R_0) = \beta_0 + 0 * \beta_1 = \beta_0$ so $R_0 = \exp[\beta_0]$ | $\ln(\text{Odds}_0) = \beta_0 + 0 * \beta_1 = \beta_0$ so $\text{Odds}_0 = \exp[\beta_0]$ |
| Risk(outcome $X_1 = 1$) | $R_1 = \beta_0 + 1 * \beta_1 = \beta_0 + \beta_1$ | $\ln(R_1) = \beta_0 + 1 * \beta_1 = \beta_0 + \beta_1$ so $R_1 = \exp[\beta_0 + \beta_1]$ | $\ln(\text{Odds}_1) = \beta_0 + 1 * \beta_1 = \beta_0 + \beta_1$ so $\text{Odds}_1 = \exp[\beta_0 + \beta_1]$ |
| Risk comparison | <p>Risk Difference</p> $\begin{aligned} \text{RD} &= R_1 - R_0 \\ &= [\beta_0 + 1 * \beta_1] - [\beta_0 + 0 * \beta_1] \\ &= [\beta_0 + \beta_1] - [\beta_0] \\ &= \beta_1 \end{aligned}$ <p>95% CI = $\beta_1 \pm (1.96 * \text{SE}(\beta_1))$</p> | <p>Risk Ratio</p> $\begin{aligned} \ln(\text{RR}) &= \ln(R_1 / R_0) = \ln(R_1) - \ln(R_0) \\ &= [\beta_0 + 1 * \beta_1] - [\beta_0 + 0 * \beta_1] \\ &= [\beta_0 + \beta_1] - [\beta_0] \\ &= \beta_1 \end{aligned}$ <p>so $\text{RR} = \exp(\beta_1)$ and 95% CI = $\exp(\beta_1 \pm (1.96 * \text{SE}(\beta_1)))$</p> | <p>Odds Ratio</p> $\begin{aligned} \ln(\text{OR}) &= \ln(O_1 / O_0) = \ln(O_1) - \ln(O_0) \\ &= [\beta_0 + 1 * \beta_1] - [\beta_0 + 0 * \beta_1] \\ &= [\beta_0 + \beta_1] - [\beta_0] \\ &= \beta_1 \end{aligned}$ <p>so $\text{OR} = \exp(\beta_1)$ and 95% CI = $\exp(\beta_1 \pm (1.96 * \text{SE}(\beta_1)))$</p> |
| R Commands | <code>glm(death ~ bord5, family = "binomial"(link = "identity"), data = dat)</code> | <code>glm(death ~ bord5, family = "binomial"(link = "log"), data = dat)</code> | <code>glm(death ~ bord5, family = "binomial"(link = "logit"), data = dat)</code> |